Given: A circle with chords $AB$ and $CD$.

Prove: $AB = CD$.

Proof:

1. Draw radii $AE$ and $CE$.
2. Since $E$ is on the circle, $AE = CE$ (radii of the same circle are equal).
3. $EA$ and $EC$ are tangents from the same point $E$.
4. Therefore, $EA = EC$.
5. Thus, $AB = CD$. 

Section B

EA and EC are tangents from outside point E. 

$EA = EC$. (1) Length of tangents from a point to a circle are equal.
Proof:

AF = AB \ (1) \ \text{length of tangents from an external point to the circle are equal}

BF = BD \ \text{length of tangents from an external point to the circle are equal}

CD = EC \ (2) \ \text{length of tangents from an external point to the circle are equal}

AB = AC \ (3) \ \text{since } AEC \text{ is a straight line}

AFBF = AE + EC \ \text{as given}

BP = EC \ \text{since } AF = AB

BD = CD \ \text{from (2) and (3)}

(1) Every number occurs in \(2, 2) (2, 4) (3, 6) (4, 2) (4, 4) (4, 6) (6, 2) (6, 4) (6, 4)

\( P \) (number of events is even) = \( \frac{9}{36} = \frac{1}{4} \)

(1) Some of outcomes are \((1, 4) (2, 3) (3, 2) (4, 1)\)
12. \( V \text{ of hemisphere} = \frac{4}{3} \pi r^3 \\
3 \pi r^2 = 462 \)
\[ 8 \times 5.7 \times r^2 = 462 \]
\[ 2 \times 49 \]
\[ r = 7 \]
\[ \text{Radius of hemisphere} = \frac{2}{3} \pi r^2 \]
\[ = \frac{2}{3} \times \frac{22}{7} \times 49 \times 7 \]
\[ = 2156/3 \]

13. The sequence goes like this,
   110, 120, 130, ..., 990
since they have a common difference of
   10. Every term can be written as
   \[ a = 110, \quad a_n = 990, \quad d = 10 \]
   \[ a_n = a + (n-1)d \]
   \[ 990 = 110 + (n-1) \times 10 \]
   \[ 980 = (n-1) \times 10 \]
   \[ 98 = (n-1) \times 10 \]
There are 89 terms between 101 and 999 divisible by 8 and 9.

\[ a = 9, \quad b = -3k, \quad c = k \]

Since roots of the equation are equal,

\[ b^2 - 4ac = 0 \]

\[ (3k)^2 - (4 \times 9 \times k) = 0 \]

\[ 9k^2 - 36k = 0 \]

\[ k^2 - 4k = 0 \]

\[ k(k - 4) = 0 \]

\[ k = 0 \quad \text{or} \quad k = 4 \]

Since k = 0 is not possible for the equation, \( k = 4 \).

\[ \angle AED = 60^\circ, \quad \angle BEC = 90^\circ \]

\[ AD = BC = 3000 \sqrt{3} \text{ m} \]

\[ \text{Let the speed of the } \]
\[ \text{aeroplane } x \times m/s \]
\[ BE = \text{ recom } DE = 30 \times X \]
\[ = 30 \times x \text{m} \]
\[ \text{(1)} \]

\[ \triangle ABE : \text{right angled} \]
\[ \tan 60^\circ = \frac{AD}{DE} \]
\[ \sqrt{3} = \frac{3000 \sqrt{3}}{DE} \]
\[ DE = 3000 \text{m} \]
\[ \text{(2)} \]

\[ \triangle DEC : \text{right angled} \]
\[ \tan 30^\circ = \frac{EC}{DE} \]
\[ \frac{1}{\sqrt{3}} = \frac{2000 \sqrt{3}}{DE} \]
\[ DE + CD = 3000 \text{m} \]
\[ 3000 + 30x = 9000 \]
\[ x = 200 \text{ m/s} \]
\[ \text{speed of plane is } 200 \text{ m/s} \]
14. side of cube, \( a = 7 \) cm

The diameter of the largest possible sphere is 7 cm. I assume as well of cube.

\[
\text{Radius} = \frac{7}{2} \times 10^{-2}
\]

Volume of the wood left = Volume of cube - Volume of sphere

\[
\begin{align*}
\text{Volume of sphere} &= \frac{4}{3} \pi \left(\frac{7}{2} \times 10^{-2}\right)^3 \\
&= \frac{4}{3} \pi \left(\frac{7}{2} \times 10^{-2}\right)^3 \\
&= \frac{4}{3} \pi \left(\frac{49}{16} \times 10^{-6}\right) \\
&= \frac{490}{48} \times 10^{-3}
\end{align*}
\]

15. Width of canal = 6 m

Height of canal = 1.5 m

Length of canal is canal in 1 h = 4 km = 4000 m

Let the Base area of field be \( x \) m²

The height of standing water in the field = \( \frac{8}{100} \) m
Volume of caeder in caerol in 10 minutes is \( \frac{1}{2} \) hour.

\[
\begin{align*}
\text{Volume of caeder in field} &= \frac{1}{2} \\
&= \frac{3.6 \times 3.5 \times 4000}{100} \\
&= \frac{3.6 \times 3.5 \times 4000}{100} \\
&= \frac{18 \times 50 \times 4000}{100} \\
&= \frac{720000}{100} \\
&= \boxed{720000} \text{ m}^3
\end{align*}
\]

10. Area of trapezium = \( 24.5 \) m²

\[
\begin{align*}
l \left( \frac{a+b}{2} \right) &= 24.5 \\
l \left( \frac{10+4}{2} \right) &= 24.5 \\
2l &= 24.5 \\
l &= \frac{24.5}{2} \\
l &= 10.25 \text{ cm}
\end{align*}
\]

\[
AB + AD = \text{given}
\]

1. \( AB \) is the length of the trapezium

\[
L = AB \times 3 \text{ cm}
\]

6. Rod AB is the radius of the quadrant.
Area of the shaded region

\[ \text{Area of quadrants} \]

\[ = \frac{34.5}{2} \times \frac{11}{3} + 6 \]

\[ = 34.5 \times 3.5 \]

\[ = 120.75 \]

\[ \frac{40 - 19.75}{2} \]

\[ = 20.125 \]

\[ \frac{39.75}{2} \]

\[ \frac{39.75}{2} \]

17. \( A(3,-3) \) and \( B(-3,7) \)

on the x-axis, the y-coordinate is zero.

so, let the point be \((x,0)\)

Let the ratio be \( k:1 \)

\[ \frac{x - 3}{-3 - 3} = \frac{-2k + 3}{k + 1} \]

\[ 7k - 3 = 0 \]

\[ k = \frac{3}{7} \]
\[
\begin{align*}
-2 + \frac{1}{3} &= x \\
\frac{7}{3} &= x \\
-2 \times 3 \frac{1}{2} + 7 &= x \\
\frac{6}{5} + \frac{3}{10} &= x \\
6 + 21 &= x \\
\frac{17}{4} &= x \\
\frac{19}{4} &= x \\
\frac{15}{7} &= x \\
\frac{16}{7} &= x \\
2x &= \frac{3}{2}
\end{align*}
\]

The coordinates of the point is \((\frac{3}{2}, 10)\)

20. Area of the shaded region = Area of major sector \(AOC\) - Area of minor sector \(AOB\)
\[ \Rightarrow \frac{360 \cdot 60}{360} \times \frac{1}{3} \times 4 \times 42 - 360 \cdot 60 \times \frac{1}{3} \times x^2 \times x \times 21 \]

\[ \Rightarrow \frac{300 \times 62}{340} \times \frac{1}{3} \times \frac{1}{4} \times 2 \times 3 \times 21 \]

\[ \Rightarrow 3465 \times 10^3 \]

\[ \frac{16}{x} - 1 = \frac{15}{x + 1} \]

\[ \frac{15}{x} - \frac{15}{x + 1} \]

\[ 16(x + 1) - 15x = x^2 + x \]

\[ 16x + 16 - 15x = x^2 + x \]

\[ 3x^2 + x^2 = 21 \]

\[ x^2 - 16 = 0 \]

\[ (x + 4)(x - 4) = 0 \]

\[ x + 4 = 0, \ x - 4 = 0 \]

\[ x = -4, \ x = 4 \]

\[ a_2 + a_7 = 30 \]
\[ a + 6d = 30 \]
\[ 3a + 7d = 30 \]  \( \cdots (1) \)

\[ (2 \times 9)^2 = 9 \]
\[ (2 \times \frac{a+7d}{2})^2 = a + 14d \]
\[ (2a + 14d)^2 = a + 14d \]
\[ 2a - 14d - 1 = a + 14d \]
\[ 2a - a = 1 \]

Substitute \( a = 1 \)
\[ 3a + 7d = 30 \]

\[ 2x + 14d = 30 \]
\[ 7d = 28 \]
\[ d = 4 \]

The A.P is 1, 5, 9, 13, 17 ...
Step 1: Construct a line segment AB of 8 cm length.
Step 2: With A as centre, draw a circle of radius 8 cm.
Step 3: With B as centre draw a circle of radius r.

Step 4: Draw a perpendicular bisector of AB and let it intersect AB at X.

Step 5: With X as centre and XA as radius, draw a circle.

Step 6: Let this circle intersect the circle with centre A at P and Q and the circle with B as centre at R and S respectively.

Step 7: Join AR, AS, BP and BQ.

AR, AS, BP and BQ are the required tangents.
24. \[ AC = \sqrt{(8 - 2)^2 + (6 - 1)^2} \]
\[ = \sqrt{3^2 + 7^2} \]
\[ = \sqrt{9 + 49} \]
\[ = \sqrt{58} \]

\[ BD = \sqrt{(8 - 2)^2 + (1 - 6)^2} \]
\[ = \sqrt{3^2 + 7^2} \]
\[ = \sqrt{9 + 49} \]
\[ = \sqrt{58} \]

Since \[ AC = BD = \sqrt{58} \text{ cm} \], the diagonals of \( ABCD \) are equal.

\[ A(2, 1) \quad B(5, 1) \quad D(2, 6) \quad C(5, 6) \]

Midpoint of \( AC \) = \( \left( \frac{3}{2}, \frac{7}{2} \right) \)

Midpoint of \( BD \) = \( \left( \frac{1}{2}, \frac{3}{2} \right) \)

Since the midpoint of diagonal \( AC \) = midpoint of
diagonal $BD = \left( \frac{1}{2}, \frac{5}{4} \right)$, which bisect each other.

SECTION D

Given: $AB$ is a tangent to circle with center $O$.

To prove: $AB$ is the longest tangent.

Proof:

1. $OC = OP = OD$ (radii of circle)

2. $OC + CT > OP$...

3. $OD = OP$

4. $OD + PR > OP$...

From (1) and (2), we can understand that $OP$ is the shortest than any distance drawn.
21. Height of tower, \( h = 24 \text{cm} \)

\[ x_1 = 8 \text{cm} \]

\[ x_2 = 8 \text{cm} \]

Volume of container = \( \frac{1}{3} \times 8 \times 8 \times \left( \frac{20 + 20 + 60 + 60}{3} \right) \times 24 \)

\[ = \frac{1}{3} \times 8 \times 8 \times \left( \frac{160 + 60 + 60}{3} \right) \times 24 \]

\[ = \frac{1}{3} \times 8 \times 8 \times \left( \frac{24 \times 24 \times 3}{1000} \right) \]

\[ = 21.6 \times 624 \times 3 \times \frac{1}{1000} \text{ litres} \]

Total cost = \( \frac{21.6 \times 624 \times 3}{1000} \times 100 \text{ Rs} \)

\[ = \frac{21.6 \times 624 \times 3}{1000} \times 100 \text{ Rs} \]

\[ = 229.47 \text{ Rs} \]

\[ \text{Ro 331.5 approximately} \]

28. Height of flagstaff = \( CD = 4 \text{m} \)

Height of tower = \( BD = 8 \text{m} \)

\( \angle DAB = 45^\circ \), \( \angle CAB = 60^\circ \)

\( AB = 120 \text{m} \)
\[ \triangle ABD \text{ is right-angled} \]

\[ \tan 45^\circ = 1 \]

\[ \frac{x}{AB} \]

\[ x = AB = 120 \text{ m} \]

\[ \triangle ACB \text{ is right-angled} \]

\[ \tan 60^\circ = \sqrt{3} \]

\[ \frac{h+x}{120} \]

\[ h + 120 = 120 \sqrt{3} \]

\[ h = 120 \sqrt{3} - 120 \]

\[ h = 120 \left( \sqrt{3} - 1 \right) \]

\[ h = 120 \left( 1.73 - 1 \right) \]

\[ h = 120 \times 0.73 \]

\[ h = 87.6 \text{ m} \]

89. Let the speed of stream = \( x \) km/h.

Then the speed of boat in upstream = \((15 - x)\) km/h.

Speed of boat in downstream = \((15 + x)\) km/h.
According to the question,

\[ 24 = 24 \]

\[ \frac{24}{15-X} \]

\[ 24 (15+X) - 24 (15-X) = 1 \]

\[ 15^2 - x^2 \]

\[ 432 + 21x - 432 + 14x = 324 - x^2 \]

\[ 21x = 324 - x^2 \]

\[ x^2 + 34x - 314 = 0 \]

\[ x^2 + 51x - 6x - 3x^2 = 0 \]

\[ x (x+51) - 6 (x+54) = 0 \]

\[ (x+51) (x-6) = 0 \]

\[ x+51 = 0 \quad x-6 = 0 \]

\[ x = -51, x = 6 \]

The speed cannot be negative.

The speed at shore is 6 kn/h.

The sequence of numbers is

\[ 2 \quad 3 \quad 1 \quad 2 \quad 12 \]

\[ 3 \quad 1 \quad 2 \quad 12 \]

\[ 3, 1, 2, 12 \]
30. The sequence of trees goes like this:

4, 8, 12, ... 48

They form an A.P. with common difference 4.

The total number of trees planted by students 1 to 12 is given by

\[ S_n = \frac{n}{2} \times (a + l) \]

where \( n = 12, \ a = 4, \ l = 48 \)

\[ S_{13} = \frac{12}{2} \times (4 + 48) \]

\[ = 6 \times 52 = 312 \]
The value of environmental conservation is shown by the students.

\[ \frac{x-3}{x-4} + \frac{x-5}{x-6} = \frac{10}{3} \]

\[ \frac{(x-3)(x-6) + (x-4)(x-5)}{(x-4)(x-6)} = \frac{10}{3} \]

\[ \Rightarrow x^2 - 9x + 18 + x^2 - 9x + 20 = 10 \]

\[ x^2 - 10x + 28 \]

\[ = 3(3x^2 - 18x + 38) = 3(10x^2 - 100x + 240) \]

\[ = (-x^2 - 54x + 180) = 103x^2 - 100x + 240 \]

\[ = x^2 - 48x + 26 = 0 \]

\[ = 2x^2 - 23x + 12 = 0 \]

\[ = (2x-9)(x-7) = 0 \]

\[ \Rightarrow 2x-9 = 0, x-7 = 0 \]

\[ \Rightarrow x = 9/2, x = 7 \]
32. 1) No. of cards remaining = 52 - 2x2
    = 52 - 4 = 48

    No. of red cards = 26 - 6 = 20
    P (red colour) = 20/48 = 5/12

    ii) No. of queens = 4 x 2
    P (a queen) = 8/48 = 1/6

    iii) No. of ace = 4
    P (any ace) = 4/48 = 1/12

    iv) No. of face cards: 12 - 6 = 6
    P (a face card) = 6/48 = 1/8

33. AD is the median of \( \triangle ABC \) (given) \\

vertex A

\[ D(x, y) = \left( \frac{3+6}{2}, \frac{-2+2}{2} \right) = (4, 0) \]
\[ \text{Area of } \triangle ADB = \frac{1}{2} \times (3 \times (0 + 1)) + 4 \times (-2 + 6) + 3 \times (-6 + 1) \]
\[ = \frac{1}{2} \times (3 + 12 - 18) \]
\[ = \frac{1}{2} \times 3 = \text{3 square units} \]  \( \text{(1)} \)

\[ \text{Area of } \triangle ACB = \frac{1}{2} \times (4 \times (0 - 2) + 4 \times (3 + 6) + 3 \times (-6 + 1)) \]
\[ = \frac{1}{2} \times (-8 + 32 - 30) \]
\[ = \frac{1}{2} \times -6 = -3 \]

Since area is positive,
\[ \text{Area of } \triangle ACB = 3 \text{ square units} \]  \( \text{(2)} \)

From (1) and (2) Area of \( \triangle ADB \) = Area of \( \triangle ACB \)

It is verified that joining \( \triangle ADB \) divides \( \triangle ABC \) into \( \triangle 100 \) triangles of equal areas.

34. Given: A circle with centre \( O \)

is inscribed in a quadrilateral \( ABCD \)

is \( \triangle AED \) and \( \triangle AFO \).

\( OE = OF \) (radii of circle)

\( \angle AFO = \angle CFA = 90^\circ \) (radii of circle)
perpendicular to the line through the point of contact is perpendicular to this tangent.

\[ \angle A = \angle A \text{ (corresponding sides)} \]

\[ \triangle AEO \cong \triangle AFO \text{ (RHS rule)} \]

\[ \angle 1 = \angle 3 \]

\[ \angle 2 = \angle 4 \]

\[ \angle 5 = \angle 6 \]

Similarly,

\[ \angle 1 = \angle 3 \]

\[ \angle 2 + \angle 4 \]

\[ \angle 5 + \angle 6 \]

\[ \angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ \]

\[ \angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ \]

\[ \angle 1 \angle 2 \angle 3 \angle 4 = 180^\circ \]

\[ \angle AOB + \angle DOC = 180^\circ \]

It is proved that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre.